1: Matrix in simple regression models

(a) Solve:

The data below show, for a consumer finance company operating in six cities, the number of competing loan companies operating in the city (X) and the number per thousand of the company's loans made in that city that are currently delinquent (Y):

Company	1	2	3	4	5
X_i	8	4	0	-4	-8
Y_i	7.8	9.0	10.2	11.0	11.7

Assume that first–order regression model is applicable. Using matrix methods:

- write the vector $\mathbf{Y}$ and the matrix $\mathbf{X}$ associated to the simple regression model
- write in matrix form (and compute) the mean of the two variables
- write in matrix form (and compute) the variance of the two variables
- write in matrix form (and compute) the covariance
- compute $\mathbf{Y}^\top\mathbf{Y}$
- compute $\mathbf{X}^\top\mathbf{X}$
- compute $\mathbf{X}^\top\mathbf{Y}$
- estimate the vector β of the LS regression
- compute the predicted values
- compute the residuals
- estimate the variance of the regression model

(b) R commands:

Check your answers to the previous exercise using R.

2: Matrix in multiple regression models

(a) Solve:

In a small–scale regression study, the following data were obtained:

i	1	2	3	4	5	6
X_{i1}	7	4	16	3	21	8
X_{i2}	33	41	7	49	5	31
Y_i	42	33	75	28	91	55

Assume that regression model with independent normal error terms is appropriate. Using matrix methods,

- write the vector $\mathbf{Y}$ and the matrix $\mathbf{X}$ associated to the multiple regression model
- write in matrix form (and compute) the mean of the two variables
- write in matrix form (and compute) the variance/covariance matrix of the two regressors
- write in matrix form (and compute) the regression coefficients
- compute the predicted values
- compute the residuals
- estimate the variance of the regression model

(b) R commands:

Check your answers to the previous exercise using R.