1: Correlation / Regression

(a) Solve:

If r, the coefficient of correlation between n pairs of values (X_i, Y_i), is positive, then determine whether each of the following statements is true or false:

- r between $(-X_i, -Y_i)$ is also positive.
- r between $(-X_i, Y_i)$ and that between $(X_i, -Y_i)$ can be either positive or negative.
- Both the slope coefficients β_{yx} and β_{xy} are positive, where β_{yx} = slope coefficient in the regression of Y on X and β_{xy} = slope coefficient in the regression of X on Y.

(b) Solve:

Let r_1 be the coefficient of correlation between n pairs of values (Y_i, X_i) and r_2 the coefficient of correlation between n pairs of values $(aX_i + b, cY_i + d)$, where a, b, c and d are constants. Show that $r_1 = r_2$ and hence establish the principle that the coefficient of correlation is invariant with respect to the change of scale and the change of origin.

(c) Solve:

In the regression $Y_i = \beta_0 + \beta_1 X_i + u_i$ suppose we multiply each X value by a constant, say, 2. Will it change the residuals and fitted values of Y? Explain. What if we add a constant value, say, 2, to each X value?

(d) Solve:

From annual observations from 1985 to 2005, the following regression results were obtained, where Y = exchange rate of the Canadian dollar to the U.S. dollar (CD/\$) and X = ratio of the U.S. consumer price index to the Canadian consumer price index; that is, X represents the relative prices in the two countries:

$$\hat{Y}_t = -0.912 + 2.250X_t, \ R^2 = 0.440, \ \text{s.e.} = 0.096$$

- Interpret this regression. How would you interpret R^2?
- Does the positive value of X_t make economic sense?
- Suppose we were to redefine X as the ratio of the Canadian CPI to the U.S. CPI. Would that change the sign of X? Why?