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Ordinal probability effect measures for dyadic analysis in cumulative models

Misure di probabilità ordinali per l'analisi diadica in modelli cumulativi

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Abstract Dyadic data analysis (DDA) is increasingly being used to better understand, analyze and model intra- and inter-personal mechanisms in various types of dyads such as husband-wife, caregiver-patient, doctor-patient, parent-child or athlete-coach as in our example. A key strength of the DDA is its flexibility to take the (non)independence available in the dyads into account. In this article, we illustrate the value of using DDA to examine how sports performance is perceived by an athlete and if it is consistent with the declared performance by his/her coach. A probability summary for ordered comparison of groups referred to a measure of stochastic superiority is used to indicate the consistency of perceived assessments.
Abstract *L'analisi dei dati diadici (DDA) è sempre più utilizzata per comprendere, analizzare e modellare i meccanismi intra e interpersonali in vari tipi di diade come marito-moglie, assistente-paziente, medico-paziente, genitore–bambino o atleta–allenatore. Quest'ultimo caso viene analizzato nel presente contributo. In particolare, la flessibilitá della DDA nel tener conto della (non)indipendenza presente nelle diadi viene qui considerata per esaminare in che modo un atleta percepisce la prestazione sportiva e se questa percezione è coerente con la valutazione della prestazione dichiarata dal suo allenatore. Al fine di considerare la coerenza delle valutazioni espresse dai due attori del processo considerato, si utilizza una misura di probabilità stocastica che permette il confronto ordinato dei gruppi.*

Key words: Dyadic analysis, Ordinal data models, Ordinal superiority measures

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1 Introduction

The study deals with modelling the actor/partner interdependence in case of categorical dyadic data by presenting an alternative approach with respect to the current used methods [7]. The research aims at evaluating the consistency of perceived assessments on some topics related to the performance in sport dyads (athletes/coaches). The proposal may adjust the score perceived by the athletes in the definition of their performance by means of coach evaluation. The analysis is about a cross–sectional study on 100 couples of athlete-coach from Italian Swimming Federation–Campania region section, collected between November and December 2019. Satisfaction and other psychological domains were measured by rating scales on a $k = 7$ point scale. Frequency distribution of some items are displayed in Figure 1.

Figure 2 is about two items, *Challenging* and *Talent*, whose response distribution of the two groups of dyads is almost the same for the first item and very different for the second one, as remarked in the next analysis. The observer agreement plot ([4], [5]) provides a graphical representation of agreements of the assessment perceived by athletes and coaches. Each chart depicts a $n \times n$ square, where n denotes the total sample size and the black squares, each of size $n_{jj} \times n_{jj}$, $j = 1, 2, \ldots, k$, denoting the observed agreement. The larger boxed rectangles depict the maximum possible agreement, since they are obtained starting from the observed marginals. Therefore, a visual impression of the strength of the agreement is obtained comparing the areas of dark squares with the area of white rectangles. A further information is provided by the lighter shaded rectangles, that depict the weighted contribution from off-diagonal cells. This allows to interpret the partial agreement among athletes and coaches comparing the areas of lighter shaded rectangles to the area of external rectangles. Finally, the positions of the dark squares with respect to the diagonal line provide information about "observer bias", i.e. the case where the observers consistently tend to classify the objects into higher or lower categories than the other.

Figure 3 depicts the path of the levels of the considered two items exploiting a correspondence analysis map [6], *Challenging* being on the left–side and *Talent* on the right–side. The two paths inside each plot represent the Athletes' levels (solid line) and the Coaches' levels (dotted line) of the two items. Multiple correspondence analysis has been carried out on the whole set of items, even if only the two illustrative items are represented for the sake of illustration. The plots confirm the different patterns of the two items, already suggested by the previous agreement plot.

2 Ordinal Superiority Measures for dyadic data analysis

When Y is a k-category ordinal response variable, one of the candidate model to analyse the rating is the cumulative link model [8]:

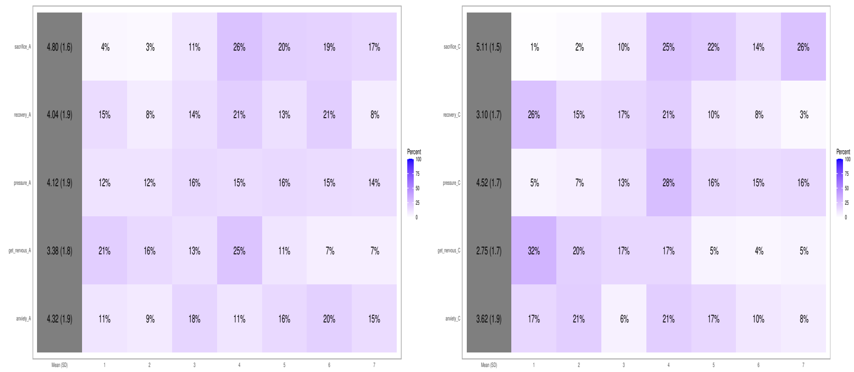

Fig. 1 Frequency distribution of subjective perceptions concerning 5 items collected on a 7 point scale (1=total disagreement; 7=total agreement). First column reports means and standard deviation. Left: Athletes' responses. Right: Coaches' responses

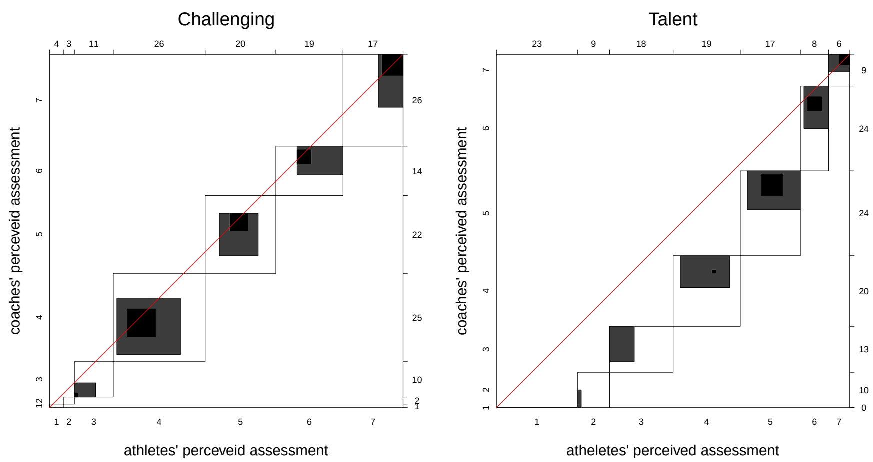

Fig. 2 Agreement plots of subjective perceptions concerning 2 items collected on a 7 point scale (1 = total disagreement; 7 = total agreement). First one is related to Athletes' challenging. Second one concerns Athletes' talent. The Athletes' responses are depicted on the horizontal axis, the Coaches' responses on the vertical axis.

$$P\left(Y_i = j \mid \boldsymbol{X}_i; \boldsymbol{\theta}\right) = P\left(\alpha_{j-1} < Y_i^* \leq \alpha_j | \boldsymbol{X}_i\right) = \\ = F_\varepsilon(\alpha_j - \boldsymbol{X}_i\boldsymbol{\beta}) - F_\varepsilon(\alpha_{j-1} - \boldsymbol{X}_i\boldsymbol{\beta}), \tag{1}$$

where

$$P\left(Y_i \leq j | \boldsymbol{X}_i; \boldsymbol{\theta}\right) = F_\varepsilon(\alpha_j - \boldsymbol{X}_i\boldsymbol{\beta}), \qquad i = 1, 2, \ldots, n; \; j = 1, 2, \ldots, k. \tag{2}$$

The parameter vector $\boldsymbol{\theta} = (\boldsymbol{\alpha}', \boldsymbol{\beta}')'$ contains the intercept values $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_{k-1})'$, where $-\infty = \alpha_0 < \alpha_1 < \ldots < \alpha_k = +\infty$ are the thresholds of the scale of a latent vari-

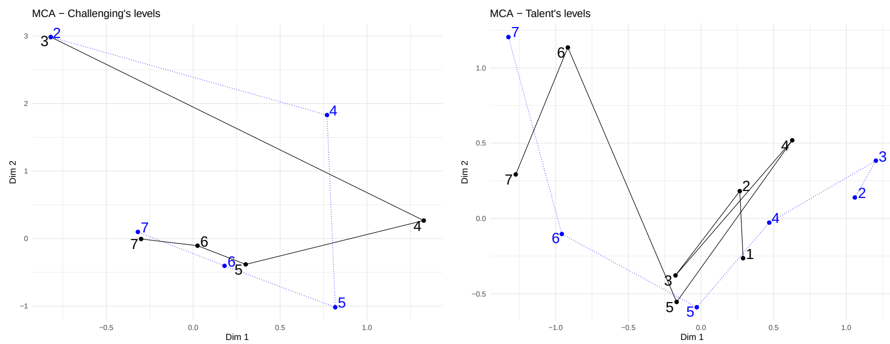

Fig. 3 Multiple correspondence maps of subjective perceptions concerning 2 items collected on a 7 point scale (1 = total disagreement; 7 = total agreement). The analysis is carried out on the whole set of items but only the levels of *Challenging* (left–side) and *Talent* (right–side) are represented. The two lines in each panel refer athletes' levels (solid black line) and coaches' levels (dotted blue line), respectively.

able Y^* surrounding the response Y, and the covariates coefficients $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)'$. The vector $\boldsymbol{X}_i = (X_{i1}, \ldots, X_{ip})$ contains instead the covariates. Here, $\boldsymbol{\theta} \in \Omega(\boldsymbol{\theta})$, where $\Omega(\boldsymbol{\theta})$ is an open subset of $\mathbb{R}^{p+k-1}$.

Some common choices for $F_\varepsilon(.)$ are the Gaussian, the logistic or the standard Gumbel distribution and the related models are named probit, logit, and extreme value, respectively. The latter, related to log-log and the complementary log-log link function, is adopted when a skewed distribution is assumed. The ordinal superiority measures apply directly to the latent variable model (see [2, 3]). These measures represent our selected approach to compare groups by studying the probability that an observation from one group (=coaches) is scored above an independent observation from the alternative group (=athletes) of dyad.

We consider a dichotomous variable to identify the two clusters of coaches and athletes. Let Y_C^* and Y_A^* be independent underlying latent response variables at $\boldsymbol{X}$. Denoting by g_C (coaches) and g_A (athletes) two groups of statistical units related to the observed Y. An "*ordinal superiority measure*" or "*measure of stochastic superiority*" is

$$\gamma = P(Y_{g_C}^* > Y_{g_A}^*; \boldsymbol{X}) = P\left[\frac{(Y_{g_C}^* - Y_{g_A}^* - \beta)}{\sqrt{2}} > \frac{-\beta}{\sqrt{2}}\right],$$

regardless of $\boldsymbol{X}$ values, which for the probit, logit and complementary loglog link [2] is, respectively:

$$\gamma = \Phi(\hat{\beta}/\sqrt{(}2)), \qquad \gamma \approx \frac{exp(\hat{\beta}/\sqrt{2})}{1+exp(\hat{\beta}/\sqrt{2})}, \qquad \gamma = \frac{exp(\hat{\beta})}{1+exp(\hat{\beta})}.$$

In the three expressions β is the group parameter. From γ index is possible to obtain the other measure

$$\Delta = 2\gamma - 1.$$

Both γ and Δ indexes are normalized ($\gamma \in [0,1]$ and $\Delta \in [-1,1]$); when $\gamma > 1/2$ or $\Delta > 0$, the ratings from g_C tend to be larger than the ratings from g_A assessing an ordinal superiority of g_C over g_A.

For γ and Δ, simple confidence intervals result directly from ordinary confidence intervals for β for the corresponding ordinal cumulative link model.

3 Results

Results point out the role played by coach adjustment enhancing the feeling and the relationship of the dyads. Table 1 reports the ordinal superiority measures along with the corresponding 95% confidence intervals for the probit model. The same results for the logit model are shown in Table 2 and for the extreme value model (complementary log–log link function) in Table 3. The value of $\hat{\gamma}$ lower than $1/2$, with exception of *pressure*, *sacrifice*, *talent*, and *work out*, indicates an ordinal inferiority of the ratings of coaches with respect to athletes. It means that coaches are less critical than their athletes. Actually, for the items *improvement* and *challenging* in the case of logit and probit models, $\hat{\Delta} \simeq 0$ providing similar responses in the two groups. Results of the latter model for an underlying extreme value distribution, which is plausible for the evaluations expressed by coaches and athletes, are quite different from the logit or probit models but lead to the same conclusions.

The ordinal superiority measures extend directly to summary comparisons of multiple groups, based on more general models that have multiple indicator variables for the groups. Thus, a possible extension may consider a triad analysis where athletes who are in teams may be analysed. In this case captain's rating may be introduced in the analysis reporting the evaluation of a peer.

Table 1 Ordinal superiority measures for probit model

	Probit model			
	$\hat{\gamma}$	$CI(\gamma)$	$\hat{\Delta}$	$CI(\Delta)$
anxiety	0.394	(0.319, 0.474)	-0.212	(-0.362, -0.052)
recovery	0.354	(0.281, 0.434)	-0.292	(-0.438, -0.132)
pressure	0.564	(0.483, 0.642)	0.128	(-0.034, 0.284)
get nervous	0.396	(0.319, 0.478)	-0.208	(-0.362, -0.044)
sacrifice	0.562	(0.481, 0.641)	0.124	(-0.038, 0.282)
challenging	0.479	(0.392, 0.568)	-0.042	(-0.216, 0.136)
talent	0.697	(0.621, 0.765)	0.394	(0.242, 0.530)
work out	0.593	(0.509, 0.673)	0.186	(0.018, 0.346)
improvement	0.477	(0.394, 0.560)	-0.046	(-0.212, 0.120)

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Table 2 Ordinal superiority measures for logit model

	Logit model			
	$\hat{\gamma}$	$CI(\gamma)$	$\hat{\Delta}$	$CI(\Delta)$
anxiety	0.388	(0.309, 0.473)	-0.224	(-0.382, -0.054)
recovery	0.345	(0.269, 0.428)	-0.310	(-0.462, -0.144)
pressure	0.563	(0.477, 0.646)	0.126	(-0.046, 0.292)
get nervous	0.386	(0.306, 0.471)	-0.228	(-0.388, -0.058)
sacrifice	0.556	(0.470, 0.640)	0.112	(-0.060, 0.280)
challenging	0.469	(0.377, 0.562)	-0.062	(-0.246, 0.124)
talent	0.703	(0.623, 0.773)	0.406	(0.246, 0.546)
work out	0.603	(0.513, 0.688)	0.206	(0.026, 0.376)
improvement	0.470	(0.381, 0.561)	-0.060	(-0.238, 0.122)

Table 3 Ordinal superiority measures for extreme value model

	Extreme value model			
	$\hat{\gamma}$	$CI(\gamma)$	$\hat{\Delta}$	$CI(\Delta)$
anxiety	0.405	(0.335, 0.478)	-0.190	(-0.330, -0.044)
recovery	0.371	(0.305, 0.442)	-0.257	(-0.390, -0.116)
pressure	0.534	(0.458, 0.608)	0.068	(-0.084, 0.216)
get nervous	0.419	(0.350, 0.491)	-0.162	(-0.300, -0.018)
sacrifice	0.557	(0.478, 0.633)	0.113	(-0.044, 0.266)
challenging	0.443	(0.347, 0.542)	-0.114	(-0.306, 0.084)
talent	0.640	(0.560, 0.696)	0.262	(0.120, 0.392)
work out	0.516	(0.433, 0.597)	0.032	(-0.134, 0.194)
improvement	0.408	(0.330, 0.490)	-0.184	(-0.340, -0.020)

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References

1. Agresti, A.: *Analysis of Ordinal Categorical Data*, 2^{nd} ed. Hoboken: Wiley (2010)
2. Agresti, A. and Kateri, M.: Ordinal Probability Effect Measures for Group Comparsons in Multinomial Cumulative Link Models. *Biometrics*, **73**, 214–219 (2017)
3. Agresti, A., and Tarantola, C.: Simple Ways to Interpret Effects in Modeling Ordinal Categorical Data, *Statistica Neerlandica*, **72**, 210–223 (2018)
4. Bangdiwala, K.: Using SAS software graphical procedures for the observer agreement chart. *Proceedings of the SAS Users Group International Conference*, 12:1083–1088 (1987)
5. Friendly, M.: *Visualizing Categorical Data*. Cary, NC: SAS Institute Inc. (2000)
6. Greenacre, M.: *Correspondence Analysis in Practice*. Third Edition. New York: Chapman and Hall/CRC (2017)
7. Kenny, D. A., Kashy, D. A., Cook, W.: *Dyadic data analysis*. New York: Guilford (2006)
8. McCullagh, P.: Regression models for ordinal data (with discussion). *Journal of the Royal Statistical Society, Series B*, **42**, 109–142 (1980)