1: The Poisson likelihood function

The Poisson r.v. is a discrete r.v. used to model the number of events occurring in a given time unit. Let X be a r.v. with density function:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

The λ parameter denotes the mean number of events occurring in the time unit.

(a) Solve:

- Formulate the expression of the likelihood function for a generic sample of n units.
- Formulate the expression of the log–likelihood function for a generic sample of n units.
- Formulate the expression of the score function (first derivative of the log–likelihood).
- Derive the maximum likelihood estimate for the λ parameter.
- Derive the Fisher observed information.

(b) R commands:

The number of road accidents observed in 10 randomly selected sunny days on the street Casilina was:

4	0	7	3	5	1	2	0	4	3

- Write an R–function for the Poisson likelihood function
- Plot the Poisson likelihood function for the given sample
- Write an R–function for the Poisson log–likelihood function
- Plot the Poisson log–likelihood function for the given sample

Another sample related to the road accidents during 20 sunny days follows:

2	4	3	3	5	7	1	1	2	2
3	1	4	2	5	1	4	2	3	1

- Graphically compare the likelihood functions for the two given samples.
- Graphically compare the log–likelihood functions for the two given samples.

2: The negative exponential likelihood function

The negative exponential r.v. is a continuous r.v. used to model the time between two subsequent events. Let X be a r.v. with density function:

$$f(x) = \lambda e^{-\lambda x}$$

The λ parameter denotes the number of events occurring in the time unit.

(a) Solve:

- Formulate the expression of the likelihood function for a generic sample of n units.
- Formulate the expression of the log–likelihood function for a generic sample of n units.
- Formulate the expression of the score function (first derivative of the log–likelihood).
- Derive the maximum likelihood estimate for the λ parameter.
- Derive the Fisher observed information.

(b) R commands:

The time (in minutes) between the arrival of a sample of 10 cars at the Cassino's tollbooth in the time slot 10:00 - 15:00 is:

0.13	0.03	0.14	0.07	0.02	0.04	0.04	0.16	0.17	0.35

- Write an R–function for the negative exponential likelihood function
- Plot the negative exponential likelihood function for the given sample
- Write an R–function for the negative exponential log–likelihood function
- Plot the negative exponential log–likelihood function for the given sample

The times for another sample of 20 cars is:

0.17	0.14	0.02	0.04	0.07	0.13	0.09	0.16	0.35	0.05
0.11	0.21	0.03	0.20	0.04	0.15	0.12	0.13	0.07	0.02

- Graphically compare the likelihood functions for the two given samples.
- Graphically compare the log–likelihood functions for the two given samples.